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TRIGONOMETRÍA

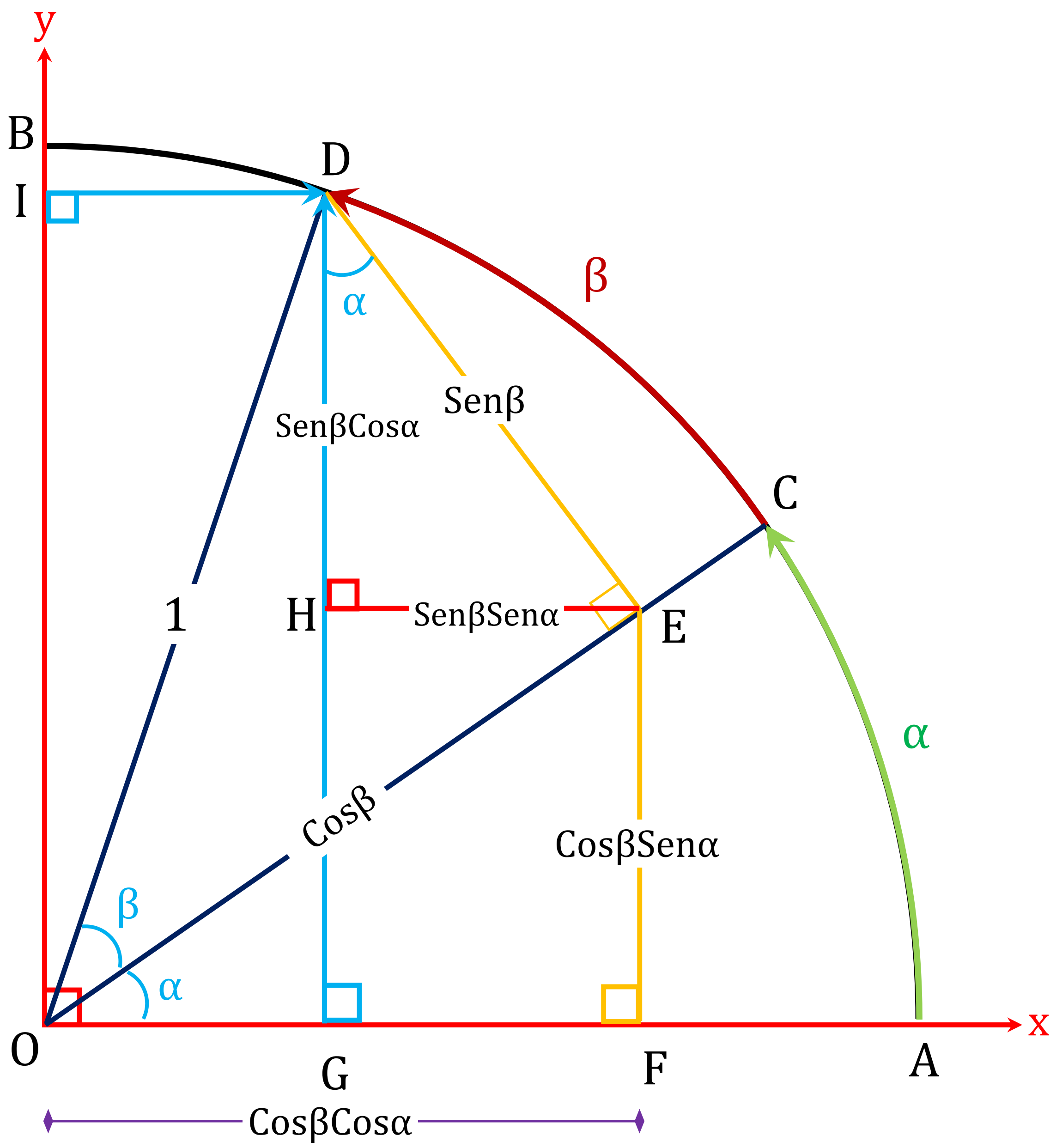
GRUPO PITÁGORAS

PITAGORAS ACADEMIA

IDENTIDADES TRIGONOMÉTRICAS

IDENTIDADES TRIGONOMÉTRICAS PARA EL ÁNGULO COMPUESTO

IDENTIDADES PARA EL ÁNGULO COMPUESTO



- $\text{Sen}(\alpha+\beta) = GD$

  $\text{Sen}(\alpha+\beta) = GH + HD$

  $\text{Sen}(\alpha+\beta) = FE + HD$

  $\mathbf{Sen(\alpha+\beta) = Sen\alpha Cos\beta + Sen\beta Cos\alpha}$

- $\text{Cos}(\alpha+\beta) = ID = OG$

  $\text{Cos}(\alpha+\beta) = OF - GF$

  $\text{Cos}(\alpha+\beta) = OF - HE$

  $\mathbf{Cos(\alpha+\beta) = Cos\alpha Cos\beta - Sen\alpha Sen\beta}$

IDENTIDADES PARA EL ÁNGULO COMPUESTO

$$\left.\begin{aligned} \text{Sen}(\alpha+\beta) &= \text{Sen}\alpha.\text{Cos}\beta + \text{Sen}\beta.\text{Cos}\alpha \\ \text{Cos}(\alpha+\beta) &= \text{Cos}\alpha.\text{Cos}\beta - \text{Sen}\alpha.\text{Sen}\beta \end{aligned}\right\} \div$$

$$\text{Tan}(\alpha+\beta) = \frac{\dfrac{\text{Sen}\alpha\text{Cos}\beta + \text{Sen}\beta\text{Cos}\alpha}{\text{Cos}\alpha\text{Cos}\beta}}{\dfrac{\text{Cos}\alpha\text{Cos}\beta - \text{Sen}\alpha\text{Sen}\beta}{\text{Cos}\alpha\text{Cos}\beta}}$$

$$\mathbf{Tan}(\boldsymbol{\alpha}+\boldsymbol{\beta}) = \frac{\mathbf{Tan}\boldsymbol{\alpha} + \mathbf{Tan}\boldsymbol{\beta}}{\mathbf{1} - \mathbf{Tan}\boldsymbol{\alpha}\mathbf{Tan}\boldsymbol{\beta}}$$

IDENTIDADES PARA EL ÁNGULO COMPUESTO

1. SUMA DE ÁNGULOS:

- $\operatorname{Sen}(x+y) = \operatorname{Sen}x.\operatorname{Cos}y + \operatorname{Sen}y.\operatorname{Cos}x$
- $\operatorname{Cos}(x+y) = \operatorname{Cos}x.\operatorname{Cos}y - \operatorname{Sen}x.\operatorname{Sen}y$
- $\operatorname{Tan}(x+y) = \dfrac{\operatorname{Tan}x + \operatorname{Tan}y}{1 - \operatorname{Tan}x.\operatorname{Tan}y}$

2. DIFERENCIA DE ÁNGULOS:

- $\text{Sen}(x - y) = \text{Senx.Cosy} - \text{Seny.Cosx}$
- $\text{Cos}(x - y) = \text{Cosx.Cosy} + \text{Senx.Seny}$
- $\text{Tan}(x - y) = \dfrac{\text{Tanx} - \text{Tany}}{1 + \text{Tanx.Tany}}$

3. IDENTIDADES AUXILIARES:

a) $\text{Sen}(x+y).\text{Sen}(x-y) = \text{Sen}^2x - \text{Sen}^2y$

***Demostración*:**

$$\text{Sen}(x+y).\text{Sen}(x-y) = \underbrace{(\text{Senx}.\text{Cosy} + \text{Seny}.\text{Cosx})}_{(a+b)}.\underbrace{(\text{Senx}.\text{Cosy} - \text{Seny}.\text{Cosx})}_{(a-b)} = a^2 - b^2$$

$$\text{Sen}(x+y).\text{Sen}(x-y) = (\text{Senx}.\text{Cosy})^2 - (\text{Seny}.\text{Cosx})^2$$

$$\text{Sen}(x+y).\text{Sen}(x-y) = \text{Sen}^2x.\text{Cos}^2y - \text{Sen}^2y.\text{Cos}^2x$$

$$\text{Sen}(x+y).\text{Sen}(x-y) = \text{Sen}^2x.(1-\text{Sen}^2y) - \text{Sen}^2y.(1-\text{Sen}^2x)$$

$$\text{Sen}(x+y).\text{Sen}(x-y) = \text{Sen}^2x - \cancel{\text{Sen}^2x.\text{Sen}^2y} - \text{Sen}^2y + \cancel{\text{Sen}^2y.\text{Sen}^2x}$$

$$\text{Sen}(x+y).\text{Sen}(x-y) = \text{Sen}^2x - \text{Sen}^2y$$

3. IDENTIDADES AUXILIARES:

b) $\text{Cos}(x+y).\text{Cos}(x-y) = \text{Cos}^2x - \text{Sen}^2y$

Demostración:

$$\text{Cos}(x+y).\text{Cos}(x-y) = \underbrace{(\text{Cosx}.\text{Cosy} - \text{Senx}.\text{Seny})}_{(a-b)}.\underbrace{(\text{Cosx}.\text{Cosy} + \text{Senx}.\text{Seny})}_{(a+b)} = a^2 - b^2$$

$$\text{Cos}(x+y).\text{Cos}(x-y) = (\text{Cosx}.\text{Cosy})^2 - (\text{Senx}.\text{Seny})^2$$

$$\text{Cos}(x+y).\text{Cos}(x-y) = \text{Cos}^2x.\text{Cos}^2y - \text{Sen}^2x.\text{Sen}^2y$$

$$\text{Cos}(x+y).\text{Cos}(x-y) = \text{Cos}^2x.(1 - \text{Sen}^2y) - \text{Sen}^2y.(1 - \text{Cos}^2x)$$

$$\text{Cos}(x+y).\text{Cos}(x-y) = \text{Cos}^2x - \cancel{\text{Cos}^2x.\text{Sen}^2y} - \text{Sen}^2y + \cancel{\text{Sen}^2y.\text{Cos}^2x}$$

$$\text{Cos}(x+y).\text{Cos}(x-y) = \text{Cos}^2x - \text{Sen}^2y$$

3. IDENTIDADES AUXILIARES:

c) $\operatorname{Tan}(x+y).\operatorname{Tan}(x-y) = \dfrac{\operatorname{Tan}^2x - \operatorname{Tan}^2y}{1-\operatorname{Tan}^2x\operatorname{Tan}^2y}$

Demostración:

$$\operatorname{Tan}(x+y).\operatorname{Tan}(x-y) = \left(\frac{\operatorname{Tan}x + \operatorname{Tan}y}{1-\operatorname{Tan}x.\operatorname{Tan}y}\right)\left(\frac{\operatorname{Tan}x - \operatorname{Tan}y}{1+\operatorname{Tan}x.\operatorname{Tan}y}\right)$$

$$\operatorname{Tan}(x+y).\operatorname{Tan}(x-y) = \frac{\operatorname{Tan}^2x - \operatorname{Tan}^2y}{1-\operatorname{Tan}^2x\operatorname{Tan}^2y}$$

3. IDENTIDADES AUXILIARES:

d) $\text{Tanx} \pm \text{Tany} = \dfrac{\text{Sen}(x \pm y)}{\text{CosxCosy}}$

Demostración:

$$\text{Tanx} \pm \text{Tany} = \frac{\text{Senx}}{\text{Cosx}} \pm \frac{\text{Seny}}{\text{Cosy}}$$

$$\text{Tanx} \pm \text{Tany} = \frac{\text{Senx}.\text{Cosy} \pm \text{Seny}.\text{Cosx}}{\text{Cosx}.\text{Cosy}}$$

$$\text{Tanx} \pm \text{Tany} = \frac{\text{Sen}(x \pm y)}{\text{CosxCosy}}$$

3. IDENTIDADES AUXILIARES:

e) $\text{Cotx} \pm \text{Coty} = \dfrac{\text{Sen}(y \pm x)}{\text{SenySenx}}$

Demostración:

$$\text{Cotx} \pm \text{Coty} = \frac{\text{Cosx}}{\text{Senx}} \pm \frac{\text{Cosy}}{Seny}$$

$$\text{Cotx} \pm \text{Coty} = \frac{\text{Seny.Cosx} \pm \text{Senx.Cosy}}{\text{Senx.Seny}}$$

$$\text{Cotx} \pm \text{Coty} = \frac{\text{Sen}(y \pm x)}{\text{SenxSeny}}$$

3. IDENTIDADES AUXILIARES:

f) $\mathrm{Tanx} \pm \mathrm{Tany} \pm \mathrm{Tan}(x \pm y).\mathrm{Tanx}.\mathrm{Tany} = \mathrm{Tan}(x \pm y)$

Demostración:

$$\mathrm{Tan}(x \pm y) = \frac{\mathrm{Tanx} \pm \mathrm{Tany}}{1 \mp \mathrm{Tanx}.\mathrm{Tany}}$$

$$\mathrm{Tan}(x \pm y)(1 \mp \mathrm{Tanx}.\mathrm{Tany}) = \mathrm{Tanx} \pm \mathrm{Tany}$$

$$\mathrm{Tan}(x \pm y) \mp \mathrm{Tan}(x \pm y).\mathrm{Tanx}.\mathrm{Tany} = \mathrm{Tanx} \pm \mathrm{Tany}$$

$$\mathrm{Tan}(x \pm y) = \mathrm{Tanx} \pm \mathrm{Tany} \pm \mathrm{Tan}(x \pm y).\mathrm{Tanx}.\mathrm{Tany}$$

3. IDENTIDADES AUXILIARES:

a) $\operatorname{Sen}(x+y)\operatorname{Sen}(x-y) = \operatorname{Sen}^2x - \operatorname{Sen}^2y$

b) $\operatorname{Cos}(x+y)\operatorname{Cos}(x-y) = \operatorname{Cos}^2x - \operatorname{Sen}^2y$

c) $\operatorname{Tan}(x+y)\operatorname{Tan}(x-y) = \dfrac{\operatorname{Tan}^2x - \operatorname{Tan}^2y}{1 - \operatorname{Tan}^2x\operatorname{Tan}^2y}$

d) $\operatorname{Tan}x \pm \operatorname{Tan}y = \dfrac{\operatorname{Sen}(x \pm y)}{\operatorname{Cos}x\operatorname{Cos}y}$

e) $\operatorname{Cot}x \pm \operatorname{Cot}y = \dfrac{\operatorname{Sen}(y \pm x)}{\operatorname{Sen}y\operatorname{Sen}x}$

f) $\operatorname{Tan}x \pm \operatorname{Tan}y \pm \operatorname{Tan}(x \pm y).\operatorname{Tan}x.\operatorname{Tan}y = \operatorname{Tan}(x \pm y)$

4. PROPIEDAD:

I) $$\frac{E}{\sqrt{a^2+b^2}} = \frac{aSenx}{\sqrt{a^2+b^2}} \pm \frac{bCosx}{\sqrt{a^2+b^2}}$$

$$\frac{E}{\sqrt{a^2+b^2}} = Senx\underbrace{\frac{a}{\sqrt{a^2+b^2}}}_{Cos\varphi} \pm Cosx\underbrace{\frac{b}{\sqrt{a^2+b^2}}}_{Sen\varphi}$$

$$\frac{E}{\sqrt{a^2+b^2}} = Senx.Cos\varphi \pm Cosx.Sen\varphi$$

$$\frac{E}{\sqrt{a^2+b^2}} = Sen(x \pm \varphi)$$

$$E = \sqrt{a^2+b^2}\,Sen(x \pm \varphi)$$

$$\left.\frac{b}{\sqrt{a^2+b^2}}\right\} Sen\varphi$$

$$\left.\frac{a}{\sqrt{a^2+b^2}}\right\} Cos\varphi$$

$$\rightarrow Tan\varphi = \frac{b}{a}$$

4. PROPIEDAD:

II) Sea: $E = a\mathrm{Sen}x + b\mathrm{Cos}x, x \in \mathbb{R}$

$$E = \sqrt{a^2 + b^2}\,\mathrm{Sen}(x + \varphi)$$

$\mathrm{Si}: x \epsilon \mathbb{R} \rightarrow (x + \varphi) \epsilon \mathbb{R}$

$-1 \leq \mathrm{Sen}(x + \varphi) \leq 1$

$\times \left(\sqrt{a^2 + b^2}\right):$

$$-\sqrt{a^2 + b^2} \leq \underbrace{\sqrt{a^2 + b^2}\,\mathrm{Sen}(x + \varphi)}_{E} \leq \sqrt{a^2 + b^2}$$

$$\underbrace{-\sqrt{a^2 + b^2}}_{E_{mín}} \leq E \leq \underbrace{\sqrt{a^2 + b^2}}_{E_{máx}}$$

4. PROPIEDADES:

I) $\mathrm{aSenx} \pm \mathrm{bCosx} = \sqrt{\mathrm{a}^2 + \mathrm{b}^2}\,\mathrm{Sen}(\mathrm{x} \pm \varphi), \mathrm{a\ y\ b} \in \mathbb{R}$

$\text{Donde: } \mathrm{Tan}\varphi = \dfrac{\mathrm{b}}{\mathrm{a}}$

II) $\text{Sea: } \mathrm{E} = \mathrm{aSenx} + \mathrm{bCosx}, \mathrm{x} \in \mathbb{R}$

$$\underbrace{-\sqrt{\mathrm{a}^2 + \mathrm{b}^2}}_{\text{mín}} \leq \mathrm{aSenx} + \mathrm{bCosx} \leq \underbrace{\sqrt{\mathrm{a}^2 + \mathrm{b}^2}}_{\text{máx}}$$

5. PROPIEDADES PARA 3 ÁNGULOS:

I) Si: $x + y + z = n\pi; n \in \mathbb{Z}$

$$x + y = n\pi - z$$

$$\mathrm{Tan}(x + y) = \mathrm{Tan}(\underbrace{n\pi - z}_{\in \mathrm{IIC} \ \vee \ \mathrm{IVC}})$$

$$\frac{\mathrm{Tanx} + \mathrm{tany}}{1 - \mathrm{Tanx.Tany}} = -\mathrm{Tanz}$$

$$\mathrm{Tanx} + \mathrm{tany} = -\mathrm{Tanz} + \mathrm{Tanx.Tany.Tanz}$$

$$\mathrm{Tanx} + \mathrm{Tany} + \mathrm{Tanz} = \mathrm{Tanx.Tany.Tanz}$$

5. PROPIEDADES PARA 3 ÁNGULOS:

II) Si: $x + y + z = n\pi; n \in \mathbb{Z}$

$$\mathrm{Tanx} + \mathrm{Tany} + \mathrm{Tanz} = \mathrm{Tanx.Tany.Tanz}$$

$$\frac{\cancel{\mathrm{Tanx}}}{\cancel{\mathrm{Tanx}}.\mathrm{Tany.Tanz}} + \frac{\cancel{\mathrm{Tany}}}{\mathrm{Tanx}.\cancel{\mathrm{Tany}}.\mathrm{Tanz}} + \frac{\cancel{\mathrm{Tanz}}}{\mathrm{Tanx.Tany}.\cancel{\mathrm{Tanz}}} = \frac{\cancel{\mathrm{Tanx.Tany.Tanz}}}{\cancel{\mathrm{Tanx.Tany.Tanz}}}$$

$$\mathrm{Cotx.Coty} + \mathrm{Cotx.Cotz} + \mathrm{Coty.Cotz} = 1$$

5. PROPIEDADES PARA 3 ÁNGULOS:

- Si: $x + y + z = n\pi; n \in \mathbb{Z}$

$$\mathrm{Tanx} + \mathrm{Tany} + \mathrm{Tanz} = \mathrm{Tanx}.\mathrm{Tany}.\mathrm{Tanz}$$

$$\mathrm{Cotx}.\mathrm{Coty} + \mathrm{Cotx}.\mathrm{Cotz} + \mathrm{Coty}.\mathrm{Cotz} = 1$$

5. PROPIEDADES PARA 3 ÁNGULOS:

I) Si: $x + y + z = (2n+1)\frac{\pi}{2}; n \in \mathbb{Z}$

$$x + y = (2n+1)\frac{\pi}{2} - z$$

$$\mathrm{Cot}(x+y) = \mathrm{Cot}\left(\underbrace{(2n+1)\frac{\pi}{2} - z}_{\in \mathrm{IC}\ \vee \mathrm{IIIC}}\right)$$

$$\frac{1 - \mathrm{Tanx.Tany}}{\mathrm{Tanx} + \mathrm{tany}} = \mathrm{Tanz}$$

$$1 - \mathrm{Tanx.Tany} = \mathrm{Tanx.Tanz} + \mathrm{Tany.Tanz}$$

$$\mathrm{Tanx.Tany} + \mathrm{Tanx.Tanz} + \mathrm{Tany.Tanz} = 1$$

5. PROPIEDADES PARA 3 ÁNGULOS:

II) Si: $x + y + z = (2n+1)\frac{\pi}{2}; n \in \mathbb{Z}$

$$\mathrm{Tanx.Tany} + \mathrm{Tanx.Tanz} + \mathrm{Tany.Tanz} = 1$$

$$\frac{\cancel{\mathrm{Tanx}}.\cancel{\mathrm{Tany}}}{\cancel{\mathrm{Tanx}}.\cancel{\mathrm{Tany}}.\mathrm{Tanz}} + \frac{\cancel{\mathrm{Tanx}}.\cancel{\mathrm{Tanz}}}{\cancel{\mathrm{Tanx}}.\mathrm{Tany}.\cancel{\mathrm{Tanz}}} + \frac{\cancel{\mathrm{Tany}}.\cancel{\mathrm{Tanz}}}{\mathrm{Tanx}.\cancel{\mathrm{Tany}}.\cancel{\mathrm{Tanz}}} = \frac{1}{\mathrm{Tanx.Tany.Tanz}}$$

$$\mathrm{Cotz} + \mathrm{Coty} + \mathrm{Cotx} = \mathrm{Cotx.Coty.Cotz}$$

5. PROPIEDADES PARA 3 ÁNGULOS :

- Si: $x + y + z = (2n + 1)\dfrac{\pi}{2};\, n \in \mathbb{Z}$

$$\mathrm{Cotx} + \mathrm{Coty} + \mathrm{Cotz} = \mathrm{Cotx}.\mathrm{Coty}.\mathrm{Cotz}$$

$$\mathrm{Tanx}.\mathrm{Tany} + \mathrm{Tanx}.\mathrm{Tanz} + \mathrm{Tany}.\mathrm{Tanz} = 1$$

MOMENTO DE PRACTICAR

PROBLEMAS Y RESOLUCIÓN

1. Si: $3(1+\text{Tan}^2\theta\text{Tan}^2\alpha) = \text{Tan}^2\theta - 8\text{Tan}\theta\text{Tan}\alpha + \text{Tan}^2\alpha$, además: $0 < \alpha < \theta < 90°$. Calcular: $\text{Cot}3\theta - \text{Cot}3\alpha$

Resolución:

$$3 + 3\text{Tan}^2\theta\text{Tan}^2\alpha = \text{Tan}^2\theta - 2\text{Tan}\theta\text{Tan}\alpha - 6\text{Tan}\theta\text{Tan}\alpha + \text{Tan}^2\alpha$$

$$3(1 + 2\text{Tan}\theta\text{Tan}\alpha + \text{Tan}^2\theta\text{Tan}^2\alpha) = \text{Tan}^2\theta - 2\text{Tan}\theta\text{Tan}\alpha + \text{Tan}^2\alpha$$

$$3(1 + \text{Tan}\theta\text{Tan}\alpha)^2 = (\text{Tan}\theta - \text{Tan}\alpha)^2$$

$$3 = \left(\frac{\text{Tan}\theta - \text{Tan}\alpha}{1 + \text{Tan}\theta\text{Tan}\alpha}\right)^2$$

$$\pm\sqrt{3} = \text{Tan}(\theta - \alpha) \longrightarrow \text{Tan}(\theta - \alpha) = \sqrt{3}$$

$$\theta - \alpha = 60° \rightarrow \theta = 60° + \alpha \rightarrow 3\theta = 180° + 3\alpha \rightarrow \text{Cot}3\theta = \text{Cot}(180° + 3\alpha)$$

$$\text{Cot}3\theta = \text{Cot}3\alpha$$

$$\therefore \mathbf{Cot3\theta - Cot3\alpha = 0}$$

CLAVE: A

IDENTIDADES PARA EL ÁNGULO COMPUESTO

2. Si: $\mathrm{Tan}\theta = \mathrm{aTan}\varphi$ (a > 0) calcular el máximo valor de: $\mathrm{Tan}(\theta - \varphi)$

Resolución:

$$\mathrm{Tan}(\theta-\varphi)=\frac{\mathrm{Tan}\theta-\mathrm{Tan}\varphi}{1+\mathrm{Tan}\theta\mathrm{Tan}\varphi}$$

$$\mathrm{Tan}(\theta-\varphi)=\frac{\mathrm{aTan}\varphi-\mathrm{Tan}\varphi}{1+\mathrm{aTan}\varphi\mathrm{Tan}\varphi}$$

$$\mathrm{Tan}(\theta-\varphi)=\frac{\mathrm{Tan}\varphi(a-1)}{1+\mathrm{aTan}^2\varphi}$$

$$\mathrm{Tan}(\theta-\varphi)=\frac{(a-1)}{\frac{1+\mathrm{aTan}^2\varphi}{\mathrm{Tan}\varphi}}$$

$$\underbrace{\mathrm{Tan}(\theta-\varphi)}_{\text{máx}}=\frac{(a-1)\ \leftarrow\ \text{cte}}{\frac{1}{\mathrm{Tan}\varphi}+\mathrm{aTan}\varphi\ \rightarrow\ \textbf{mín}}$$

$$\mathrm{aTan}\varphi+\frac{1}{\mathrm{Tan}\varphi}\geq 2\sqrt{\mathrm{a}\cancel{\mathrm{Tan}\varphi}\frac{1}{\cancel{\mathrm{Tan}\varphi}}}$$

$$\mathrm{aTan}\varphi+\frac{1}{\mathrm{Tan}\varphi}\geq 2\sqrt{a}$$

Reemplazando:

$$\therefore\ \mathbf{Tan(\theta-\varphi)=\frac{a-1}{2\sqrt{a}}}$$

CLAVE: C

3. Calcular $\frac{Sen\theta}{Sen\alpha}$, si: $xCot\theta - yCot\alpha = (x - y)Cot\left(\frac{\theta+\alpha}{2}\right)$

Resolución:

$$xCot\theta - yCot\alpha = xCot\left(\frac{\theta+\alpha}{2}\right) - yCot\left(\frac{\theta+\alpha}{2}\right)$$

$$x\underbrace{\left[Cot\theta - Cot\left(\frac{\theta+\alpha}{2}\right)\right]} = y\underbrace{\left[Cot\alpha - Cot\left(\frac{\theta+\alpha}{2}\right)\right]}$$

$$x\left[\frac{Sen\left(\frac{\theta+\alpha}{2}-\theta\right)}{\cancel{Sen\left(\frac{\theta+\alpha}{2}\right)}Sen\theta}\right] = y\left[\frac{Sen\left(\frac{\theta+\alpha}{2}-\alpha\right)}{\cancel{Sen\left(\frac{\theta+\alpha}{2}\right)}Sen\alpha}\right]$$

$$x\left[\frac{\cancel{Sen\left(\frac{\alpha-\theta}{2}\right)}}{Sen\theta}\right] = y\left[\frac{\overset{-1}{\cancel{Sen\left(\frac{\theta-\alpha}{2}\right)}}}{Sen\alpha}\right]$$

$$\therefore \frac{Sen\theta}{Sen\alpha} = -\frac{x}{y}$$

CLAVE: C

IDENTIDADES PARA EL ÁNGULO COMPUESTO

4. Si: $2\text{Tan}\theta - \text{Cot}\alpha = \text{Tan}\beta$; además: $\dfrac{\text{Tan}\theta\text{Tan}\alpha}{\text{Tan}\varphi\text{Tan}\beta} = \dfrac{\text{Cos}(\alpha-\beta)}{\text{Cos}(\alpha+\beta)}$. Calcular : $\text{Cot}\beta - \text{Tan}\alpha$

Resolución:

$$2\text{Tan}\theta - \frac{1}{\text{Tan}\alpha} = \text{Tan}\beta$$

$$2\text{Tan}\theta\text{Tan}\alpha - 1 = \text{Tan}\alpha\text{Tan}\beta$$

$$2\text{Tan}\theta\text{Tan}\alpha = 1 + \text{Tan}\alpha\text{Tan}\beta$$

$$\frac{\text{Tan}\theta\text{Tan}\alpha}{\text{Tan}\varphi\text{Tan}\beta} = \frac{\dfrac{\text{Cos}\alpha\text{Cos}\beta+\text{Sen}\alpha\text{Sen}\beta}{\text{Cos}\alpha\text{Cos}\beta}}{\dfrac{\text{Cos}\alpha\text{Cos}\beta-\text{Sen}\alpha\text{Sen}\beta}{\text{Cos}\alpha\text{Cos}\beta}}$$

$$\frac{\text{Tan}\theta\text{Tan}\alpha}{\text{Tan}\varphi\text{Tan}\beta} = \frac{1+\text{Tan}\alpha\text{Tan}\beta}{1-\text{Tan}\alpha\text{Tan}\beta}$$

$$\frac{\cancel{\text{Tan}\theta\text{Tan}\alpha}}{\text{Tan}\varphi\text{Tan}\beta} = \frac{2\cancel{\text{Tan}\theta\text{Tan}\alpha}}{1-\text{Tan}\alpha\text{Tan}\beta}$$

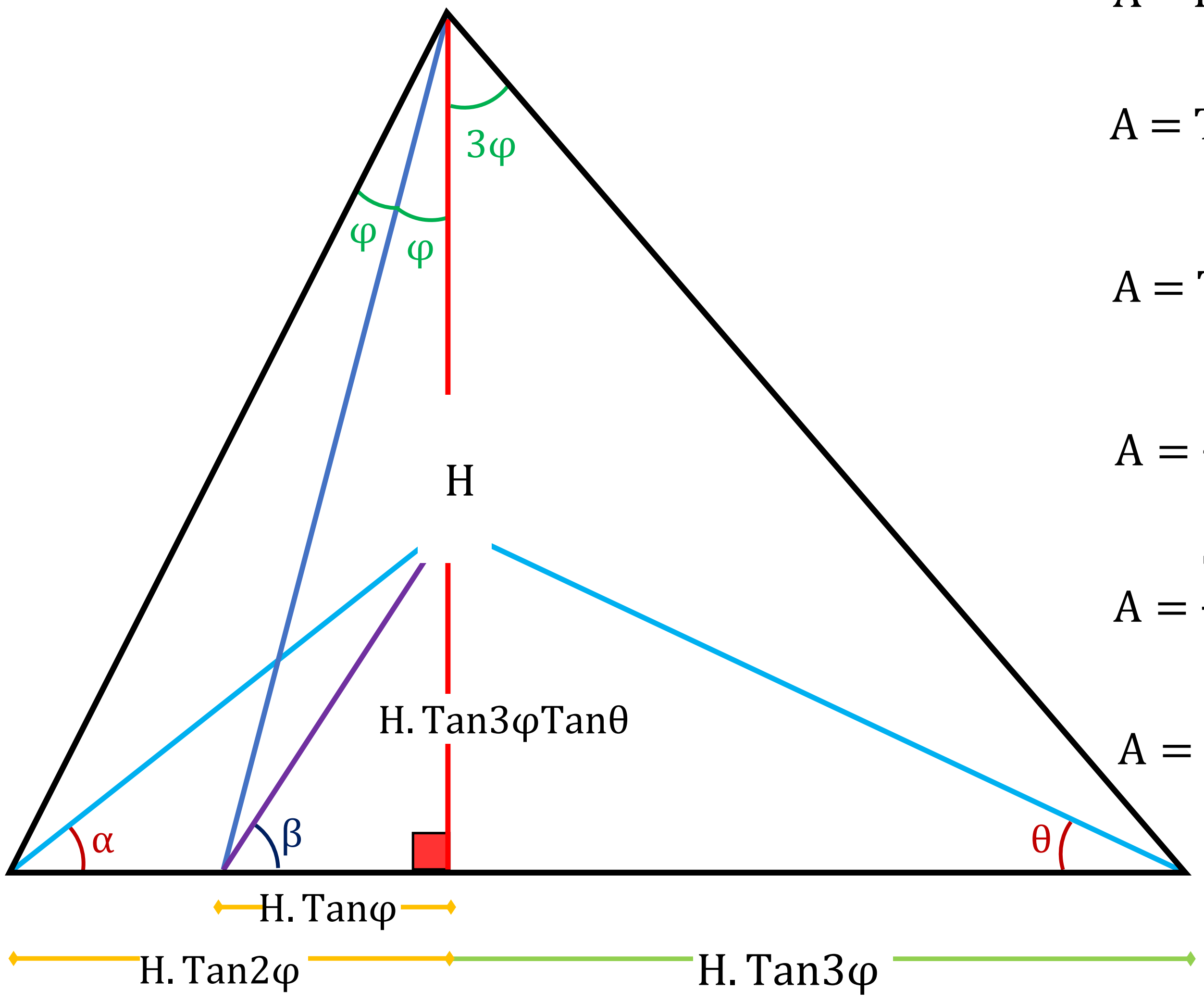
$$\frac{1-\text{Tan}\alpha\text{Tan}\beta}{\text{Tan}\beta} = 2\text{Tan}\varphi$$

$$\therefore\ \mathbf{Cot\beta - Tan\alpha = 2Tan\varphi}$$

CLAVE: C

IDENTIDADES PARA EL ÁNGULO COMPUESTO

5. Calcular : $\dfrac{\text{Tan}2\varphi}{\text{Cot}\varphi}+\dfrac{\text{Tan}\theta}{\text{Tan}\alpha}+\dfrac{\text{Tan}\theta}{\text{Tan}\beta}$



Resolución:

$$A=\text{Tan}2\varphi\text{Tan}\varphi+\text{Tan}\theta\text{Cot}\alpha+\text{Tan}\theta\text{Cot}\beta$$

$$A=\text{Tan}2\varphi\text{Tan}\varphi+\cancel{\text{Tan}\theta}\,\frac{\cancel{H}\text{Tan}2\varphi}{\cancel{H}\text{Tan}3\varphi\cancel{\text{Tan}\theta}}+\cancel{\text{Tan}\theta}\,\frac{\cancel{H}\text{Tan}\varphi}{\cancel{H}\text{Tan}3\varphi\cancel{\text{Tan}\theta}}$$

$$A=\text{Tan}2\varphi\text{Tan}\varphi+\frac{\text{Tan}2\varphi}{\text{Tan}3\varphi}+\frac{\text{Tan}\varphi}{\text{Tan}3\varphi}$$

$$A=\frac{\text{Tan}2\varphi\text{Tan}\varphi\text{Tan}3\varphi+\text{Tan}2\varphi+\text{Tan}\varphi}{\text{Tan}3\varphi}$$

$$A=\frac{\text{Tan}2\varphi+\text{Tan}\varphi+\text{Tan}(2\varphi+\varphi)\text{Tan}2\varphi\text{Tan}\varphi}{\text{Tan}3\varphi}$$

$$A=\frac{\cancel{\text{Tan}3\varphi}}{\cancel{\text{Tan}3\varphi}}$$

$\therefore$ **A = 1**

CLAVE: E

6. Si: A, B, C son los ángulos internos de un triángulo, se cumple:

$$\tan^2\varphi\tan A = \tan B + \tan C \qquad \cot^2\theta\tan B = \tan A + \tan C \qquad \tan^2\alpha\tan C = \tan A + \tan B$$

calcular: $\dfrac{\mathrm{Sen}(\varphi+\theta)\mathrm{Sen}(\varphi-\theta)}{1-\mathrm{Sen}^2\alpha}$

$A + B + C = 180°$

$\boxed{\begin{array}{l}\tan A + \tan B + \tan C = \tan A\tan B\tan C \\ \cot A\cot B + \cot A\cot C + \cot B\cot C = 1\end{array}}$

Resolución:

$$B = \frac{\mathrm{Sen}^2\varphi - \mathrm{Sen}^2\theta}{\mathrm{Cos}^2\alpha}$$

$$B = \frac{(1-\mathrm{Cos}^2\varphi)-(\mathrm{Sen}^2\theta)}{\mathrm{Cos}^2\alpha}$$

Reemplazando:

$$B = \frac{1 - \cot B\cot C - \cot A\cot C}{\cot A\cot B}$$

$$B = \frac{\cancel{\cot A\cot B}}{\cancel{\cot A\cot B}}$$

$\therefore \mathbf{B = 1}$

- $\tan A + \tan^2\varphi\tan A = \tan A + \tan B + \tan C$

  $\sec^2\varphi\cancel{\tan A} = \cancel{\tan A}.\tan B.\tan C$

  AL^{-1} $\qquad \cos^2\varphi = \cot B.\cot C$

- $\tan B + \cot^2\theta\tan B = \tan B + \tan A + \tan C$

  $\csc^2\theta\cancel{\tan B} = \tan A.\cancel{\tan B}.\tan C$

  AL^{-1} $\qquad \mathrm{Sen}^2\theta = \cot A.\cot C$

- $\tan C + \tan^2\alpha\tan C = \tan C + \tan A + \tan B$

  $\sec^2\alpha\cancel{\tan C} = \tan A.\tan B.\cancel{\tan C}$

  AL^{-1} $\qquad \cos^2\alpha = \cot A.\cot B$

CLAVE: C

7. Si:

$$\text{Tan}\alpha\text{Tan}\theta + \text{Tan}\alpha\text{Tan}\beta + \text{Tan}\theta\text{Tan}\beta = 1 \quad \wedge \quad \text{Tan}(\alpha\text{-}x)+\text{Tan}(\theta\text{-}x)+\text{Tan}(\beta\text{-}x)=\text{Tan}(\alpha\text{-}x)\text{Tan}(\beta\text{-}x)\text{Tan}(\theta\text{-}x)$$

calcular el mayor valor de x, sabiendo que: $x \in \,]0; 2\pi[$

Resolución:

$$\text{Tan}\alpha\text{Tan}\theta + \text{Tan}\alpha\text{Tan}\beta + \text{Tan}\theta\text{Tan}\beta = 1$$

$$\rightarrow \alpha + \beta + \theta = \frac{(2n+1)\pi}{2}, n \in \mathbb{Z}$$

$$\text{Tan}(\alpha-x)+\text{Tan}(\theta-x)+\text{Tan}(\beta-x)=\text{Tan}(\alpha-x)\text{Tan}(\beta-x)\text{Tan}(\theta-x)$$

$$\rightarrow \alpha-x+\beta-x+\theta-x = k\pi, k \in \mathbb{Z}$$

$$\alpha+\beta+\theta - 3x = k\pi$$

Reemplazando:

$$n\pi + \frac{\pi}{2} - 3x = k\pi$$

$$3x = (n-k)\pi + \frac{\pi}{2}$$

$$x = \frac{(n-k)\pi}{3} + \frac{\pi}{6}$$

$$\therefore x = \frac{11\pi}{6}$$

CLAVE: C

8. Si : a=Cos1°+3Cos3°+5Cos5°+7Cos7°+... "n" términos
   b=Cos91°+3Cos93°+5Cos95°+7Cos97°+..."n términos"
calcular: Cos46°+3Cos48°+5Cos50°+7Cos52°+...."n términos"

Resolución:

$$b = \mathrm{Cos}(90°+1°)+3\mathrm{Cos}(90°+3°)+5\mathrm{Cos}(90°+5°)+7\mathrm{Cos}(90°+7°)+\ldots$$

$$b = -\mathrm{Sen}1° - 3\mathrm{Sen}3° - 5\mathrm{Sen}5° - 7\mathrm{Sen}7° - \ldots$$

$$a = \mathrm{Cos}1° + 3\mathrm{Cos}3° + 5\mathrm{Cos}5° + 7\mathrm{Cos}7° + \ldots$$

$+$

$$a+b = \mathrm{Cos}1° - \mathrm{Sen}1° + 3(\mathrm{Cos}3° - \mathrm{Sen}3°) + 5(\mathrm{Cos}5° - \mathrm{Sen}5°) + 7(\mathrm{Cos}7° - \mathrm{Sen}7°) + \ldots$$

$$\frac{a+b}{\sqrt{2}} = \underbrace{\frac{1}{\sqrt{2}}}_{\mathrm{Cos}45°}\mathrm{Cos}1° - \underbrace{\frac{1}{\sqrt{2}}}_{\mathrm{Sen}45°}\mathrm{Sen}1° + 3\left(\underbrace{\frac{1}{\sqrt{2}}}_{\mathrm{Cos}45°}\mathrm{Cos}3° - \underbrace{\frac{1}{\sqrt{2}}}_{\mathrm{Sen}45°}\mathrm{Sen}3°\right) + 5\left(\underbrace{\frac{1}{\sqrt{2}}}_{\mathrm{Cos}45°}\mathrm{Cos}5° - \underbrace{\frac{1}{\sqrt{2}}}_{\mathrm{Sen}45°}\mathrm{Sen}5°\right) + 7\left(\underbrace{\frac{1}{\sqrt{2}}}_{\mathrm{Cos}45°}\mathrm{Cos}7° - \underbrace{\frac{1}{\sqrt{2}}}_{\mathrm{Sen}45°}\mathrm{Sen}7°\right) + \ldots$$

$$\frac{a+b}{\sqrt{2}} = \mathrm{Cos}(45°+1°) + 3\mathrm{Cos}(45°+3°) + 5\mathrm{Cos}(45°+5°) + 7\mathrm{Cos}(45°+7°) + \ldots$$

$$\therefore\ \mathbf{Cos46°+3Cos48°+5Cos50°+7Cos52°+} \cdots = \frac{\sqrt{2}}{2}(\mathbf{a}+\mathbf{b})$$

CLAVE: E

9. Si: $\text{Tan}(\alpha - \theta) = \text{Tan}^3\theta$, calcular: $\dfrac{\text{Tan}\alpha\text{Cos}2\theta}{\text{Sen}\theta\text{Cos}\theta}$

Resolución:

$$\frac{\text{Tan}\alpha - \text{Tan}\theta}{1 + \text{Tan}\alpha\text{Tan}\theta} = \text{Tan}^3\theta$$

$$\text{Tan}\alpha - \text{Tan}\theta = \text{Tan}^3\theta(1 + \text{Tan}\alpha\text{Tan}\theta)$$

$$\text{Tan}\alpha - \text{Tan}\theta = \text{Tan}^3\theta + \text{Tan}\alpha\text{Tan}^4\theta$$

$$\text{Tan}\alpha - \text{Tan}\alpha\text{Tan}^4\theta = \text{Tan}^3\theta + \text{Tan}\theta$$

$$\text{Tan}\alpha(1 - \text{Tan}^4\theta) = \text{Tan}\theta(\text{Tan}^2\theta + 1)$$

$$\text{Tan}\alpha\cancel{(1 + \text{Tan}^2\theta)}(1 - \text{Tan}^2\theta) = \text{Tan}\theta\cancel{(\text{Tan}^2\theta + 1)}$$

$$2\,\text{Tan}\alpha = \frac{2\text{Tan}\theta}{1 - \text{Tan}^2\theta}$$

$$\mathbf{2Tan\alpha = Tan2\theta}$$

$$V = \frac{\text{Tan}\alpha\text{Cos}2\theta}{\text{Sen}\theta\text{Cos}\theta}$$

$$V = \frac{2\text{Tan}\alpha\text{Cos}2\theta}{2\text{Sen}\theta\text{Cos}\theta}$$

$$V = \frac{2\text{Tan}\alpha\text{Cos}2\theta}{\text{Sen}2\theta}$$

$$V = \underbrace{2\text{Tan}\alpha}\text{Cot}2\theta$$

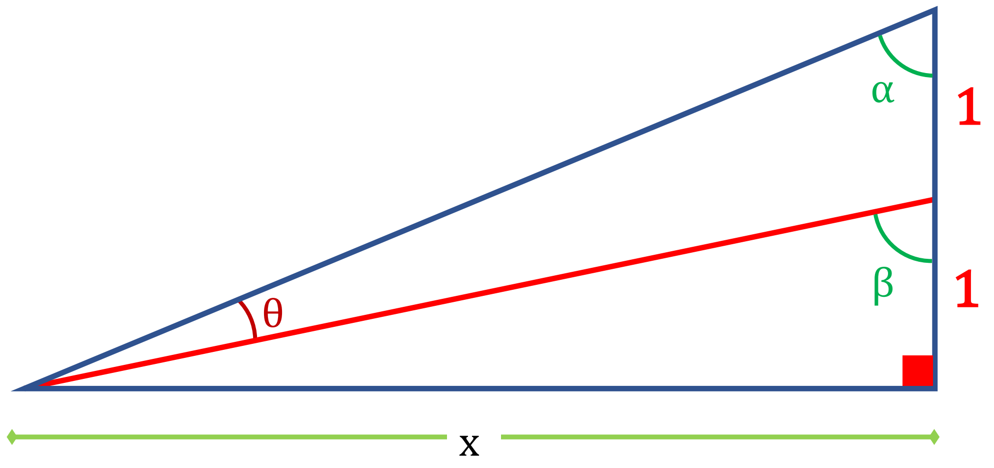
$$V = \text{Tan}2\theta\text{Cot}2\theta$$

$$\therefore \mathbf{V = 1}$$

CLAVE: E

IDENTIDADES PARA EL ÁNGULO COMPUESTO

10. Hallar x para que el ángulo θ sea máximo



Resolución:

$$\theta + \alpha = \beta$$

$$\theta = \beta - \alpha$$

$$\text{Tan}\beta = \frac{x}{1} \qquad \text{Tan}\alpha = \frac{x}{2}$$

"Tanθ" es máximo

$$\text{Tan}\theta = \text{Tan}(\beta - \alpha)$$

$$\text{Tan}\theta = \frac{\text{Tan}\beta - \text{Tan}\alpha}{1 + \text{Tan}\beta.\text{Tan}\alpha}$$

$$\text{Tan}\theta = \frac{x - \frac{x}{2}}{1 + x.\frac{x}{2}}$$

$$\text{Tan}\theta = \frac{\frac{x}{\cancel{2}}}{\frac{2+x^2}{\cancel{2}}}$$

$$\text{Tan}\theta = \frac{1}{\frac{2+x^2}{x}}$$

$$\underbrace{\text{Tan}\theta}_{\text{máx}} = \frac{1 \leftarrow \text{cte}}{\frac{2}{x} + x \rightarrow \text{mín}}$$

$$\text{Sea: } E = \frac{2}{x} + x$$

Si es $E_{\text{mínimo}}$

$$\rightarrow \frac{2}{x} = x$$

$$x^2 = 2$$

$$\therefore \mathbf{x = \sqrt{2}}$$

CLAVE: B

Obs:

$$\mathbf{Tan\theta_{máx}} = \frac{\mathbf{1}}{\mathbf{2\sqrt{2}}}$$

IDENTIDADES PARA EL ÁNGULO COMPUESTO

11. Si : $x\text{Cos}\theta + y\text{Sen}\theta = b$

$$x\text{Cos}(\theta+\varphi) + y\text{Sen}(\theta+\varphi) = 4a$$
$$x\text{Cos}(\theta - \varphi) + y\text{Sen}(\theta - \varphi) = 2a$$

Calcular: $\text{Cos}\varphi$

Resolución:

- $x(\text{Cos}\theta\text{Cos}\varphi - \text{Sen}\theta\text{Sen}\varphi) + y(\text{Sen}\theta\text{Cos}\varphi + \text{Sen}\varphi\text{Cos}\theta) = 4a$

$$x\text{Cos}\theta\text{Cos}\varphi - \cancel{x\text{Sen}\theta\text{Sen}\varphi} + y\text{Sen}\theta\text{Cos}\varphi + \cancel{y\text{Sen}\varphi\text{Cos}\theta} = 4a$$

- $x(\text{Cos}\theta\text{Cos}\varphi + \text{Sen}\theta\text{Sen}\varphi) + y(\text{Sen}\theta\text{Cos}\varphi - \text{Sen}\varphi\text{Cos}\theta) = 2a$

$$x\text{Cos}\theta\text{Cos}\varphi + \cancel{x\text{Sen}\theta\text{Sen}\varphi} + y\text{Sen}\theta\text{Cos}\varphi - \cancel{y\text{Sen}\varphi\text{Cos}\theta} = 2a$$

$+$

$$\cancel{2}x\text{Cos}\theta\text{Cos}\varphi + \cancel{2}y\text{Sen}\theta\text{Cos}\varphi = \cancel{6}a$$

$$\text{Cos}\varphi\underbrace{(x\text{Cos}\theta + y\text{Sen}\theta)}_{b} = 3a$$

$$\therefore \text{Cos}\varphi = \frac{3a}{b}$$

CLAVE: D

12. Si: $\text{Tan}\alpha - \text{Cot}(\alpha+\beta) = a$ y $\text{Sec}\alpha - \text{Csc}(\alpha+\beta) = b$, hallar en términos de a y b: $\dfrac{\text{Cos}\alpha - \text{Sen}(\alpha+\beta)}{\text{Cos}(2\alpha+\beta)}$

Resolución:

$$\frac{\text{Sen}\alpha}{\text{Cos}\alpha} - \frac{\text{Cos}(\alpha+\beta)}{\text{Sen}(\alpha+\beta)} = a$$

$$\frac{\text{Sen}\alpha.\text{Sen}(\alpha+\beta) - \text{Cos}\alpha.\text{Cos}(\alpha+\beta)}{\text{Cos}\alpha.\text{Sen}(\alpha+\beta)} = a$$

$$\frac{-\text{Cos}(2\alpha+\beta)}{\text{Cos}\alpha.\text{Sen}(\alpha+\beta)} = a$$

$$\frac{1}{\text{Cos}\alpha} - \frac{1}{\text{Sen}(\alpha+\beta)} = b$$

$$\frac{\text{Sen}(\alpha+\beta) - \text{Cos}\alpha}{\text{Cos}\alpha.\text{Sen}(\alpha+\beta)} = b$$

$$\frac{-\left(\text{Cos}\alpha - \text{Sen}(\alpha+\beta)\right)}{\text{Cos}\alpha.\text{Sen}(\alpha+\beta)} = b$$

$\div$

$$\frac{\cancel{-}\left(\text{Cos}\alpha - \text{Sen}(\alpha+\beta)\right)}{\cancel{-}\text{Cos}(2\alpha+\beta)} = \frac{b}{a}$$

CLAVE: B

IDENTIDADES PARA EL ÁNGULO COMPUESTO

13. Acerca de la expresión : $\mathrm{Cos}^2x - \dfrac{2\mathrm{CosxCosa}}{\mathrm{Sec}(a+x)} + 1 - \mathrm{Sen}^2(a+x)$, se afirma que:

Resolución:

$$\mathrm{Cos}^2x - 2\mathrm{CosxCosaCos}(a+x) + \mathrm{Cos}^2(a+x)$$

$$\mathrm{Cos}^2x - \mathrm{Cos}(a+x)[2\mathrm{CosxCosa} - \mathrm{Cos}(a+x)]$$

$$\mathrm{Cos}^2x - \mathrm{Cos}(a+x)[2\mathrm{CosxCosa} - (\mathrm{CosxCosa} - \mathrm{SenxSena})]$$

$$\mathrm{Cos}^2x - \mathrm{Cos}(a+x)[\underbrace{\mathrm{CosxCosa} + \mathrm{SenxSena}}]$$

$$\mathrm{Cos}^2x - \underbrace{\mathrm{Cos}(a+x)[\mathrm{Cos}(x-a)]}$$

$$\mathrm{Cos}^2x - [\mathrm{Cos}^2x - \mathrm{Sen}^2a]$$

$$\mathbf{Sen^2a}$$

CLAVE: D

IDENTIDADES PARA EL ÁNGULO COMPUESTO

14. Si: $\dfrac{\text{Sen}\alpha}{\text{Sen}(\alpha+\beta)} = n$, calcular : $\text{Cos}\beta - \dfrac{\text{Sen}\beta}{\text{Tan}(\alpha+\beta)} + (1 + \text{Tan}2x\text{Tan}x)\text{Sen}\left(\dfrac{\pi}{2} - 2x\right)$

Resolución:

$$W = \text{Cos}\beta - \text{Sen}\beta\text{Cot}(\alpha+\beta) + (1 + \text{Tan}2x\text{Tan}x)\text{Cos}2x$$

$$\text{Sec}2x - 1 = \text{Tan}2x\text{Tan}x$$

$$W = \text{Cos}\beta - \text{Sen}\beta\frac{\text{Cos}(\alpha+\beta)}{\text{Sen}(\alpha+\beta)} + (\text{Sec}2x)\text{Cos}2x$$

$$W = \underbrace{\frac{\text{Sen}(\alpha+\beta)\text{Cos}\beta - \text{Sen}\beta\text{Cos}(\alpha+\beta)}{\text{Sen}(\alpha+\beta)}} + 1$$

$$W = \underbrace{\frac{\text{Sen}(\alpha+\cancel{\beta} - \cancel{\beta})}{\text{Sen}(\alpha+\beta)}} + 1$$

$$\therefore \mathbf{W = n + 1}$$

CLAVE: C

IDENTIDADES PARA EL ÁNGULO COMPUESTO

15. Simplificar la siguiente sumatoria que tiene "n" sumandos :
$\text{Sec}\alpha\text{Sec}(\alpha+\beta)+\text{Sec}(\alpha+\beta)\text{Sec}(\alpha+2\beta)+\ldots$

$$\text{Tan}x - \text{Tan}y = \frac{\text{Sen}(x-y)}{\text{Cos}x\text{Cos}y}$$

Resolución:

$$L = \text{Sec}\alpha\text{Sec}(\alpha+\beta)+\text{Sec}(\alpha+\beta)\text{Sec}(\alpha+2\beta)+\cdots + \text{Sec}(\alpha+(n-1)\beta)\text{Sec}(\alpha+n\beta)$$

$$L = \frac{1}{\text{Cos}\alpha\text{Cos}(\alpha+\beta)} + \frac{1}{\text{Cos}(\alpha+\beta)\text{Cos}(\alpha+2\beta)} + \cdots + \frac{1}{\text{Cos}(\alpha+(n-1)\beta)\text{Cos}(\alpha+n\beta)}$$

$+\beta$ $\quad$ $+\beta$ $\quad$ $+\beta$

$$L\text{Sen}\beta = \underbrace{\frac{\text{Sen}((\alpha+\beta)-\alpha)}{\text{Cos}\alpha\text{Cos}(\alpha+\beta)}}_{\text{Tan}(\alpha+\beta)-\text{Tan}\alpha} + \underbrace{\frac{\text{Sen}\big((\alpha+2\beta)-(\alpha+\beta)\big)}{\text{Cos}(\alpha+\beta)\text{Cos}(\alpha+2\beta)}}_{\text{Tan}(\alpha+2\beta)-\text{Tan}(\alpha+\beta)} + \cdots + \underbrace{\frac{\text{Sen}\big((\alpha+n\beta)-(\alpha+(n-1)\beta)\big)}{\text{Cos}(\alpha+(n-1)\beta)\text{Cos}(\alpha+n\beta)}}_{\text{Tan}(\alpha+n\beta)-\text{Tan}(\alpha+(n-1)\beta)}$$

IDENTIDADES PARA EL ÁNGULO COMPUESTO

$$\left.\begin{array}{l}
\cancel{\mathrm{Tan}(\alpha+\beta)} - \mathrm{Tan}\alpha \\
\cancel{\mathrm{Tan}(\alpha+2\beta)} - \cancel{\mathrm{Tan}(\alpha+\beta)} \\
\cancel{\mathrm{Tan}(\alpha+3\beta)} - \cancel{\mathrm{Tan}(\alpha+2\beta)} \\
\cancel{\mathrm{Tan}(\alpha+4\beta)} - \cancel{\mathrm{Tan}(\alpha+3\beta)} \\
\quad\vdots \qquad\qquad\qquad \vdots \\
\mathrm{Tan}(\alpha+n\beta) - \cancel{\mathrm{Tan}(\alpha+(n-1)\beta)}
\end{array}\right\} +$$

$$\mathrm{LSen}\beta = \mathrm{Tan}(\alpha+n\beta) - \mathrm{Tan}\alpha$$

$$\therefore\ \mathbf{L = Csc\beta(Tan(\alpha+n\beta) - Tan\alpha)}$$

CLAVE: B o E

IDENTIDADES PARA EL ÁNGULO COMPUESTO

16. En un ΔABC se cumple: $\mathrm{Cos}A = \mathrm{Sen}x\mathrm{Cos}y$, $\mathrm{Cos}B = \mathrm{Sen}y\mathrm{Cos}z$, $\mathrm{Cos}C = \mathrm{Sen}z\mathrm{Cos}x$
calcular : $\mathrm{Tan}x\mathrm{Tan}y\mathrm{Tan}z$

Resolución:

$$A + B + C = 180°$$

$$\mathrm{Cos}A = -\mathrm{Cos}(B + C)$$

$$\mathrm{Cos}A = -\mathrm{Cos}B\mathrm{Cos}C + \mathrm{Sen}B\mathrm{Sen}C$$

$$\mathrm{Cos}A + \mathrm{Cos}B\mathrm{Cos}C = \mathrm{Sen}B\mathrm{Sen}C$$

AL^2

$$\mathrm{Cos}^2A + 2\mathrm{Cos}A\mathrm{Cos}B\mathrm{Cos}C + \mathrm{Cos}^2B\mathrm{Cos}^2C = \mathrm{Sen}^2B\mathrm{Sen}^2C$$

$$\mathrm{Cos}^2A + 2\mathrm{Cos}A\mathrm{Cos}B\mathrm{Cos}C + \mathrm{Cos}^2B\mathrm{Cos}^2C = (1 - \mathrm{Cos}^2B)(1 - \mathrm{Cos}^2C)$$

$$\mathrm{Cos}^2A + 2\mathrm{Cos}A\mathrm{Cos}B\mathrm{Cos}C + \cancel{\mathrm{Cos}^2B\mathrm{Cos}^2C} = 1 - \mathrm{Cos}^2B - \mathrm{Cos}^2C + \cancel{\mathrm{Cos}^2B\mathrm{Cos}^2C}$$

$$\mathrm{Cos}^2A + \mathrm{Cos}^2B + \mathrm{Cos}^2C + 2\mathrm{Cos}A\mathrm{Cos}B\mathrm{Cos}C = 1$$

$$\boxed{\textbf{Si: } A + B + C = 180° \rightarrow \mathrm{Cos}^2A + \mathrm{Cos}^2B + \mathrm{Cos}^2C + 2\mathrm{Cos}A\mathrm{Cos}B\mathrm{Cos}C = 1}$$

IDENTIDADES PARA EL ÁNGULO COMPUESTO

16. En un ΔABC se cumple: $\text{CosA} = \text{SenxCosy}$, $\text{CosB} = \text{SenyCosz}$, $\text{CosC} = \text{SenzCosx}$, calcular : TanxTanyTanz

Resolución:

$$\text{Cos}^2\text{A} + \text{Cos}^2\text{B} + \text{Cos}^2\text{C} + 2\text{CosACosBCosC} = 1$$

Reemplazando:

$$\frac{\text{Sen}^2\text{xCos}^2\text{y} + \text{Sen}^2\text{yCos}^2\text{z} + \text{Sen}^2\text{zCos}^2\text{x}}{\text{Cos}^2\text{xCos}^2\text{yCos}^2\text{z}} + \frac{2\text{SenxSenySenz}\cancel{\text{Cosx}}\cancel{\text{Cosy}}\cancel{\text{Cosz}}}{\text{CosxCosyCosz}.\cancel{\text{Cosx}}\cancel{\text{Cosy}}\cancel{\text{Cosz}}} = \frac{1}{\text{Cos}^2\text{xCos}^2\text{yCos}^2\text{z}}$$

$$\text{Tan}^2\text{x}.\text{Sec}^2\text{z} + \text{Tan}^2\text{y}.\text{Sec}^2\text{x} + \text{Tan}^2\text{z}.\text{Sec}^2\text{y} + 2\text{TanxTanyTanz} = \text{Sec}^2\text{x}.\text{Sec}^2\text{y}.\text{Sec}^2\text{z}$$

$$\text{Tan}^2\text{x}(1 + \text{Tan}^2\text{z}) + \text{Tan}^2\text{y}(1 + \text{Tan}^2\text{x}) + \text{Tan}^2\text{z}(1 + \text{Tan}^2\text{y}) + 2\text{TanxTanyTanz} = (1 + \text{Tan}^2\text{x})(1 + \text{Tan}^2\text{y})(1 + \text{Tan}^2\text{z})$$

$$\cancel{\text{Tan}^2\text{x}} + \cancel{\text{Tan}^2\text{xTan}^2\text{z}} + \cancel{\text{Tan}^2\text{y}} + \cancel{\text{Tan}^2\text{xTan}^2\text{y}} + \cancel{\text{Tan}^2\text{z}} + \cancel{\text{Tan}^2\text{yTan}^2\text{z}} + 2\text{TanxTanyTanz} = 1 + \cancel{\text{Tan}^2\text{x}} + \cancel{\text{Tan}^2\text{y}} + \cancel{\text{Tan}^2\text{z}} + \cancel{\text{Tan}^2\text{xTan}^2\text{z}} + \cancel{\text{Tan}^2\text{xTan}^2\text{y}} + \cancel{\text{Tan}^2\text{yTan}^2\text{z}} + \text{Tan}^2\text{xTan}^2\text{yTan}^2\text{z}$$

$$2\text{TanxTanyTanz} = 1 + \text{Tan}^2\text{xTan}^2\text{yTan}^2\text{z}$$

$$0 = 1 - 2\text{TanxTanyTanz} + \text{Tan}^2\text{xTan}^2\text{yTan}^2\text{z}$$

$$(\text{TanxTanyTanz} - 1)^2 = 0$$

$$\therefore \textbf{TanxTanyTanz} = \mathbf{1}$$

CLAVE: A

16. En un ΔABC se cumple: $\text{Cos}A = \text{Cos}\alpha\text{Cos}\beta$, $\text{Cos}B = \text{Cos}\beta\text{Cos}\gamma$, $\text{Cos}C = \text{Cos}\gamma\text{Cos}\alpha$
calcular : $\text{Tan}\alpha\text{Tan}\beta\text{Tan}\theta$

Resolución:

$$\text{Cos}^2A + \text{Cos}^2B + \text{Cos}^2C + 2\text{Cos}A\text{Cos}B\text{Cos}C = 1$$

Reemplazando:

$$\frac{\text{Cos}^2\alpha\text{Cos}^2\beta + \text{Cos}^2\beta\text{Cos}^2\gamma + \text{Cos}^2\gamma\text{Cos}^2\alpha}{\text{Cos}^2\alpha\text{Cos}^2\beta\text{Cos}^2\gamma} + \frac{2\text{Cos}^2\alpha\text{Cos}^2\beta\text{Cos}^2\gamma}{\text{Cos}^2\alpha\text{Cos}^2\beta\text{Cos}^2\gamma} = \frac{1}{\text{Cos}^2\alpha\text{Cos}^2\beta\text{Cos}^2\gamma}$$

$$\text{Sec}^2\alpha + \text{Sec}^2\beta + \text{Sec}^2\gamma + 2 = \text{Sec}^2\alpha.\text{Sec}^2\beta.\text{Sec}^2\gamma$$

$$(1 + \text{Tan}^2\alpha) + (1 + \text{Tan}^2\beta) + (1 + \text{Tan}^2\gamma) + 2 = (1 + \text{Tan}^2\alpha)(1 + \text{Tan}^2\beta)(1 + \text{Tan}^2\theta)$$

$$\cancel{1 + \text{Tan}^2\alpha} + \cancel{1 + \text{Tan}^2\beta} + \cancel{1 + \text{Tan}^2\gamma} + 2 = \cancel{1 + \text{Tan}^2\alpha + \text{Tan}^2\beta + \text{Tan}^2\gamma} + \text{Tan}^2\alpha\text{Tan}^2\gamma + \text{Tan}^2\alpha\text{Tan}^2\beta + \text{Tan}^2\beta\text{Tan}^2\gamma + \text{Tan}^2\alpha\text{Tan}^2\beta\text{Tan}^2\gamma$$

$$4 = \text{Tan}^2\alpha\text{Tan}^2\gamma + \text{Tan}^2\alpha\text{Tan}^2\beta + \text{Tan}^2\beta\text{Tan}^2\gamma + \text{Tan}^2\alpha\text{Tan}^2\beta\text{Tan}^2\gamma$$

IDENTIDADES PARA EL ÁNGULO COMPUESTO

17. Si x, y son valores distintos de θ que satisfacen la ecuación: $a\mathrm{Tan}\theta + b\mathrm{Sec}\theta = 1$. Hallar: $\mathrm{Tan}(x+y)$

Resolución:

$$b\mathrm{Sec}\theta = 1 - a\mathrm{Tan}\theta$$

AL^2

$$b^2\mathrm{Sec}^2\theta = 1 - 2a\mathrm{Tan}\theta + a^2\mathrm{Tan}^2\theta$$

$$b^2(1+\mathrm{Tan}^2\theta) = 1 - 2a\mathrm{Tan}\theta + a^2\mathrm{Tan}^2\theta$$

$$(a^2-b^2)\mathrm{Tan}^2\theta - 2a\mathrm{Tan}\theta + 1 - b^2 = 0 \quad \begin{cases} \mathrm{Tan}\theta_1 = \mathrm{Tan}x \\ \mathrm{Tan}\theta_2 = \mathrm{Tan}y \end{cases}$$

Cardano:

$$\mathrm{Tan}x + \mathrm{Tan}y = -\frac{-2a}{(a^2-b^2)}$$

$$\mathrm{Tan}x.\mathrm{Tan}y = \frac{1-b^2}{(a^2-b^2)}$$

$$\mathrm{Tan}(x+y) = \frac{\mathrm{Tan}x + \mathrm{Tan}y}{1 - \mathrm{Tan}x\mathrm{Tan}y}$$

$$\mathrm{Tan}(x+y) = \frac{\frac{2a}{(a^2-b^2)}}{1 - \frac{(1-b^2)}{(a^2-b^2)}}$$

$$\therefore \mathbf{Tan(x+y)} = \frac{\mathbf{2a}}{\mathbf{a^2-1}}$$

CLAVE: B

IDENTIDADES PARA EL ÁNGULO COMPUESTO

18. Sabiendo que: $a\text{Sen}x + b\text{Sen}y + c\text{Sen}z = 0$ y $a\text{Cos}x + b\text{Cos}y + c\text{Cos}z = 0$, calcular : $\dfrac{\text{Sen}(x-z)}{\text{Sen}(y-z)}$

Resolución:

- $a\text{Sen}x + b\text{Sen}y = -c\text{Sen}z$

$$\frac{a\text{Sen}x + b\text{Sen}y}{\text{Sen}z} = -c$$

- $a\text{Cos}x + b\text{Cos}y = -c\text{Cos}z$

$$\frac{a\text{Cos}x + b\text{Cos}y}{\text{Cos}z} = -c$$

$$\frac{a\text{Sen}x + b\text{Sen}y}{\text{Sen}z} = \frac{a\text{Cos}x + b\text{Cos}y}{\text{Cos}z}$$

$$a\text{Sen}x\text{Cos}z + b\text{Sen}y\text{Cos}z = a\text{Sen}z\text{Cos}x + b\text{Sen}z\text{Cos}y$$

$$a(\text{Sen}x\text{Cos}z - \text{Sen}z\text{Cos}x) = -b(\text{Sen}y\text{Cos}z - \text{Sen}z\text{Cos}y)$$

$$a\text{Sen}(x-z) = -b\text{Sen}(y-z)$$

$$\therefore \frac{\text{Sen}(x-z)}{\text{Sen}(y-z)} = -\frac{b}{a}$$

CLAVE: D

IDENTIDADES PARA EL ÁNGULO COMPUESTO

19. Si se cumple :Sen^2y - $Sen^2z=Tan^2xCos(y - z)Cos(y+z)$, calcular: $Tan(x - y)\ Tan(x + y) + Tan^2z$

Resolución:

$$\frac{Sen(y+z)Sen(y-z)}{Cos(y+z)Cos(y-z)} = Tan^2x$$

$$Tan(y+z)Tan(y-z) = Tan^2x$$

$$\frac{Tan^2y - Tan^2z}{1 - Tan^2yTan^2z} = Tan^2x$$

$$Tan^2y - Tan^2z = Tan^2x(1 - Tan^2yTan^2z)$$

$$Tan^2y - Tan^2z = Tan^2x - Tan^2xTan^2yTan^2z$$

$$Tan^2xTan^2yTan^2z - Tan^2z = Tan^2x - Tan^2y$$

$$-Tan^2z(1 - Tan^2xTan^2y) = Tan^2x - Tan^2y$$

$$-Tan^2z = \frac{Tan^2x - Tan^2y}{1 - Tan^2xTan^2y}$$

$$-Tan^2z = Tan(x+y)Tan(x-y)$$

$$Tan(x+y)Tan(x-y) + Tan^2z = 0$$

CLAVE: C

IDENTIDADES PARA EL ÁNGULO COMPUESTO

20. Si: $\mathrm{Tan}5\theta + \mathrm{Tan}\theta = a$ y $\mathrm{Cot}\theta + \mathrm{Cot}5\theta = b$, calcular: $\dfrac{\mathrm{Sen}6\theta\mathrm{Sen}5\theta\mathrm{Sen}\theta}{\mathrm{Cos}^2\theta\mathrm{Cos}^2 5\theta}$ en términos de a y b

Resolución:

$$D = \frac{\mathrm{Sen}6\theta\mathrm{Sen}5\theta\mathrm{Sen}\theta}{\mathrm{Cos}5\theta\mathrm{Cos}\theta\mathrm{Cos}5\theta\mathrm{Cos}\theta}$$

$$D = \underbrace{\frac{\mathrm{Sen}6\theta}{\mathrm{Cos}5\theta\mathrm{Cos}\theta}}\mathrm{Tan}5\theta\mathrm{Tan}\theta$$

$$D = (\underbrace{\mathrm{Tan}5\theta + \mathrm{Tan}\theta}_{a})\mathrm{Tan}5\theta\mathrm{Tan}\theta$$

$$D = a\mathrm{Tan}5\theta\mathrm{Tan}\theta$$

Reemplazando:

$$D = a \times \frac{a}{b}$$

$$\therefore \mathbf{D = \frac{a^2}{b}}$$

$$\mathrm{Cot}\theta + \mathrm{Cot}5\theta = b$$

$$\frac{1}{\mathrm{Tan}\theta} + \frac{1}{\mathrm{Tan}5\theta} = b$$

$$\frac{\mathrm{Tan}5\theta + \mathrm{Tan}\theta}{\mathrm{Tan}5\theta\mathrm{Tan}\theta} = b$$

$$\frac{a}{\mathrm{Tan}5\theta\mathrm{Tan}\theta} = b$$

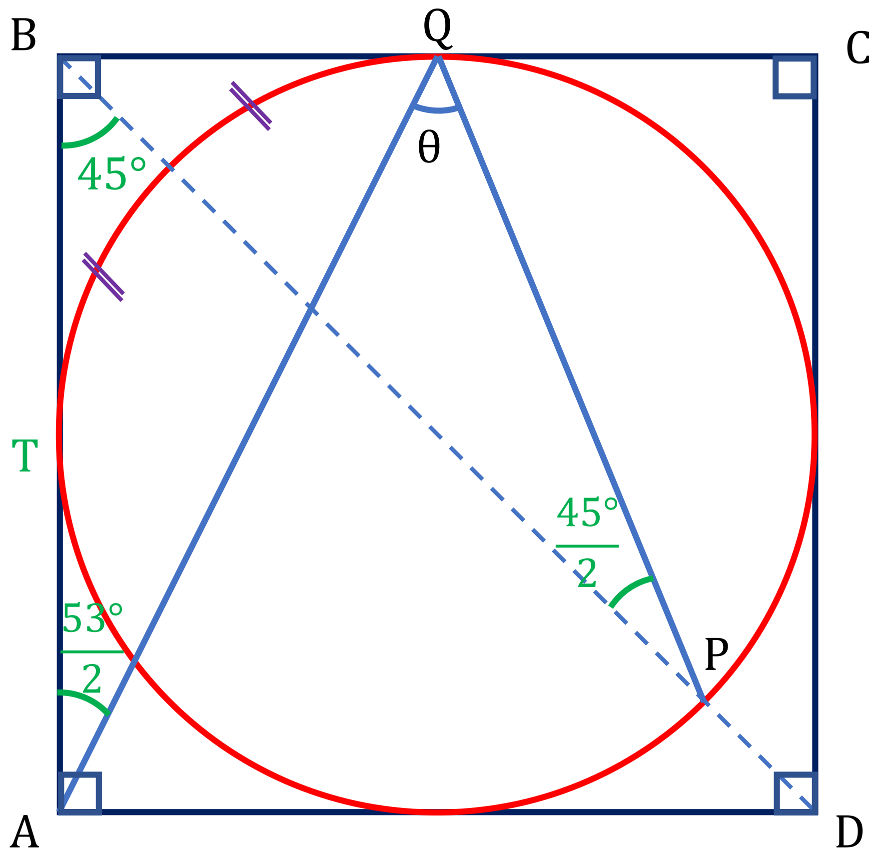
$$\mathrm{Tan}5\theta\mathrm{Tan}\theta = \frac{a}{b}$$

CLAVE: B

UNI 2019 – I:

En la figura mostrada, ABCD es un cuadrado. Si $\overline{BD}$ corta a la circunferencia inscrita en P y Q es un punto de tangencia, calcule Tanθ.

A) $\frac{2\sqrt{2}-1}{5}$ B) $\frac{3\sqrt{2}-1}{3}$ C) $\frac{5\sqrt{2}+1}{7}$ D) $\frac{2\sqrt{2}+1}{4}$ E) $\frac{3\sqrt{2}+1}{5}$



Resolución:

- Del gráfico:

$$\theta + \frac{45°}{2} = 45° + \frac{53°}{2}$$

$$\theta = \frac{45°}{2} + \frac{53°}{2}$$

$$\text{Tan}\theta = \text{Tan}\left(\frac{45°}{2} + \frac{53°}{2}\right) = \frac{(\sqrt{2}-1) + \frac{1}{2}}{1 - (\sqrt{2}-1)\frac{1}{2}}$$

$$\text{Tan}\theta = \frac{(2\sqrt{2}-1)}{(3-\sqrt{2})}\,\frac{(3+\sqrt{2})}{(3+\sqrt{2})}$$

$$\therefore\ \textbf{Tan}\theta = \frac{\mathbf{5\sqrt{2}+1}}{\mathbf{7}}$$

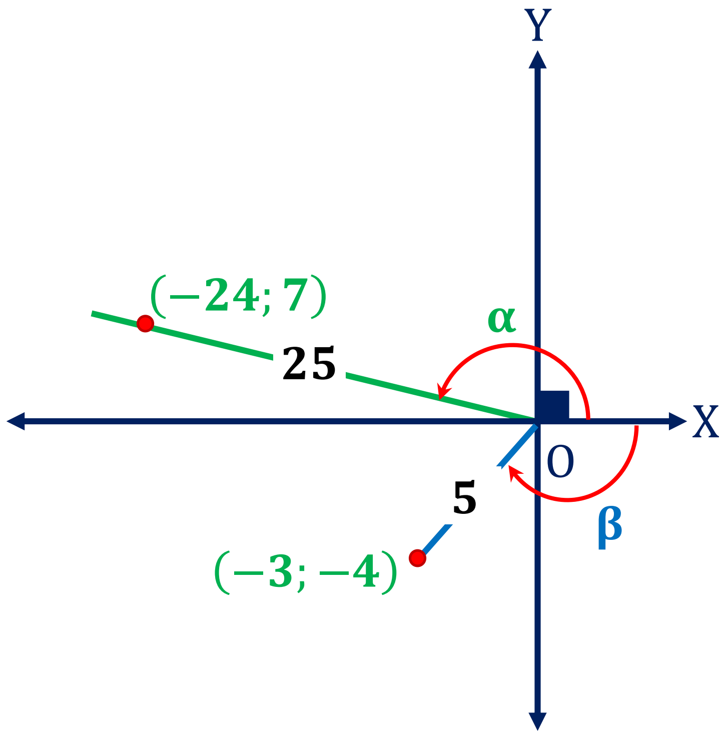
Clave: C

UNI 2019 – I:

Sea α un ángulo en el II cuadrante con $\text{Tan}\alpha = -\frac{7}{24}$ y β un ángulo en el III cuadrante con $\text{Cot}\beta = \frac{3}{4}$ Determine el valor de Sen(α + β).

A) $-\frac{107}{125}$ B) $-\frac{3}{5}$ C) $\frac{17}{125}$ D) $\frac{3}{5}$ E) $\frac{107}{125}$

Resolución:



$$\text{Sen}(\alpha + \beta)$$

$$\text{Sen}\alpha\text{Cos}\beta + \text{Sen}\beta\text{Cos}\alpha$$

$$\frac{7}{25}\cdot\frac{-3}{5} + \frac{-4}{5}\cdot\frac{-24}{25}$$

$$\frac{-21 + 96}{125}$$

$$\frac{3}{5}$$

Clave: D

UNI 2018 – I:

Simplifique: $K = \sqrt{3(\text{Cot}60° + \text{Tan}27°)(\text{Cot}60° + \text{Tan}33°)}$

A) 1 B) 2 C) 3 D) 4 E) 5

Resolución:

$$K = \sqrt{3(\text{Tan}30° + \text{Tan}27°)(\text{Tan}30° + \text{Tan}33°)}$$

$$K = \sqrt{3\left(\frac{\cancel{\text{Sen}(57°)}}{\text{Cos}30°.\cancel{\text{Cos}27°}}\right)\left(\frac{\cancel{\text{Sen}(63°)}}{\text{Cos}30°.\cancel{\text{Cos}33°}}\right)}$$

$$K = \sqrt{3\left(\frac{1}{\text{Cos}^2 30°}\right)}$$

$$K = \sqrt{\cancel{3}\left(\frac{4}{\cancel{3}}\right)}$$

$\therefore$ **K = 2**

Clave: B

UNI 2016 – I:

Sean x, y, z las medidas de los ángulos interiores de un triángulo tales que: $\text{Cotx} + \text{Coty} = 3\text{Tanz}.\text{Cotx}.\text{Coty}$

Determine Tanx en función del ángulo y.

A) 2Tany B) 3Cosy C) 4Coty D) 3Tany E) 4Seny

Resolución:

$$\text{Cotx} + \text{Coty} = 3\text{Tanz}.\text{Cotx}.\text{Coty}$$

$$\frac{\cancel{\text{Cotx}}}{\cancel{\text{Cotx}}.\text{Coty}} + \frac{\cancel{\text{Coty}}}{\text{Cotx}.\cancel{\text{Coty}}} = 3\text{Tanz}$$

$$\underbrace{\text{Tanz} + \text{Tany} + \text{Tanx}}_{\text{Tanx}.\text{Tany}.\text{Tanz}} = 3\text{Tanz} + \text{Tanz}$$

$$\text{Tanx}.\text{Tany}.\cancel{\text{Tanz}} = 4\cancel{\text{Tanz}}$$

$$\therefore \textbf{Tanx} = \textbf{4Coty}$$

Clave: C

UNI 2015 – I:

Al simplificar la expresión: $K=\left[\operatorname{Cos}^2\left(\frac{\pi}{3}+x\right)-\operatorname{Cos}^2\left(\frac{\pi}{3}-x\right)-\frac{\sqrt{3}}{2}\right](1-\operatorname{Sen}2x)$, se obtiene:

A) $-\frac{\sqrt{3}}{2}\operatorname{Cos}^2 2x$ B) $\frac{\sqrt{3}}{2}\operatorname{Sen}^2 2x$ C) $-\frac{\sqrt{3}}{2}\operatorname{Sec}2x$ D) $\frac{\sqrt{3}}{2}\operatorname{Csc}x$ E) $\frac{\sqrt{3}}{2}$

Resolución:

$$K=\left[\operatorname{Cos}^2\left(\frac{\pi}{3}+x\right)-\operatorname{Cos}^2\left(\frac{\pi}{3}-x\right)-\frac{\sqrt{3}}{2}\right](1-\operatorname{Sen}2x)$$

$$K=\left[\cancel{1}-\operatorname{Sen}^2(60°+x)-(\cancel{1}-\operatorname{Sen}^2(60°-x))-\frac{\sqrt{3}}{2}\right](1-\operatorname{Sen}2x)$$

$$K=\left[\operatorname{Sen}^2(60°-x)-\operatorname{Sen}^2(60°+x)-\frac{\sqrt{3}}{2}\right](1-\operatorname{Sen}2x)$$

$$K=\left[\operatorname{Sen}120°.\operatorname{Sen}(-2x)-\frac{\sqrt{3}}{2}\right](1-\operatorname{Sen}2x)$$

$$K=\left[-\frac{\sqrt{3}}{2}\operatorname{Sen}2x-\frac{\sqrt{3}}{2}\right](1-\operatorname{Sen}2x)$$

$$K=-\frac{\sqrt{3}}{2}(\operatorname{Sen}2x+1)(1-\operatorname{Sen}2x)$$

$$K=-\frac{\sqrt{3}}{2}(1-\operatorname{Sen}^2 2x)$$

$$\therefore \mathbf{K=-\frac{\sqrt{3}}{2}\operatorname{Cos}^2 2x}$$

Clave: A

UNI 2012 – I:

Si $\operatorname{Tan}(x(k+y)) = a$ y $\operatorname{Tan}(x(k-y)) = b$, entonces $\operatorname{Tan}(2kx) + \operatorname{Tan}(2yx)$ es igual a:

A) $\frac{a^2-b^2}{1+a^2b^2}$ B) $\frac{a^2-b^2}{1-a^2b^2}$ C) $\frac{a^2+b^2}{1+a^2b^2}$ D) $\frac{2a(1+b^2)}{1+a^2b^2}$ E) $\frac{2a(1+b^2)}{1-a^2b^2}$

Resolución:

$$\operatorname{Tan}(\underbrace{xk+xy}_{\theta}) = a$$

$$\operatorname{Tan}(\underbrace{xk-xy}_{\varphi}) = b$$

$$xk + xy = \theta$$

$$xk - xy = \varphi$$

$$(+):\ 2xk = \theta + \varphi$$

$$(-):\ 2xy = \theta - \varphi$$

$$\operatorname{Tan}(\theta+\varphi) + \operatorname{Tan}(\theta-\varphi)$$

$$\frac{a+b}{1-ab} + \frac{a-b}{1+ab}$$

$$\frac{(1+ab)(a+b) + (1-ab)(a-b)}{(1-ab)(1+ab)}$$

$$\frac{a + \cancel{b} + a^2b + \cancel{ab^2} + a - \cancel{b} - \cancel{a^2b} + ab^2}{1-a^2b^2}$$

$$\frac{2a(1+b^2)}{1-a^2b^2}$$

Clave: E

IDENTIDADES PARA EL ÁNGULO COMPUESTO

UNI 2011 – II:

Si $\operatorname{Tan}\left(\frac{4x}{7}\right) = a$ y $\operatorname{Tan}\left(\frac{3x}{7}\right) = b$, entonces al simplificar: $E = (1 - a^2b^2).\operatorname{Tan}x.\operatorname{Tan}\left(\frac{x}{7}\right)$; se obtiene:

A) $a - b$ B) $a^2 - b^2$ C) $a + b$ D) ab E) $\frac{a}{b}$

Resolución:

$$\operatorname{Tan}\left(\frac{4x}{7}\right) = \operatorname{Tan}\left(\frac{4x}{7} + \frac{3x}{7}\right)$$

$$\operatorname{Tan}\left(\frac{x}{7}\right) = \operatorname{Tan}\left(\frac{4x}{7} - \frac{3x}{7}\right)$$

Por I. Auxiliar:

$$\operatorname{Tan}\left(\frac{4x}{7} + \frac{3x}{7}\right).\operatorname{Tan}\left(\frac{4x}{7} - \frac{3x}{7}\right) = \frac{\operatorname{Tan}^2\left(\frac{4x}{7}\right) - \operatorname{Tan}^2\left(\frac{x}{7}\right)}{1 - \operatorname{Tan}^2\left(\frac{4x}{7}\right)\operatorname{Tan}^2\left(\frac{x}{7}\right)}$$

$$\operatorname{Tan}x.\operatorname{Tan}\left(\frac{x}{7}\right) = \frac{a^2 - b^2}{1 - a^2b^2}$$

$$(1 - a^2b^2).\operatorname{Tan}x.\operatorname{Tan}\left(\frac{x}{7}\right) = a^2 - b^2$$

$$\therefore \mathbf{E = a^2 - b^2}$$

Clave: B

UNI 2011 – I:

En un triángulo acutángulo ABC. Calcule el valor de: $E = \dfrac{\operatorname{Cos}(A-B)}{\operatorname{Sen}A.\operatorname{Sen}B} + \dfrac{\operatorname{Cos}(B-C)}{\operatorname{Sen}B.\operatorname{Sen}C} + \dfrac{\operatorname{Cos}(A-C)}{\operatorname{Sen}A.\operatorname{Sen}C}$

A) 3 B) 4 C) 5 D) 6 E) 8

Resolución:

$$E = \frac{\operatorname{Cos}A.\operatorname{Cos}B + \operatorname{Sen}A.\operatorname{Sen}B}{\operatorname{Sen}A.\operatorname{Sen}B} + \frac{\operatorname{Cos}B.\operatorname{Cos}C + \operatorname{Sen}B.\operatorname{Sen}C}{\operatorname{Sen}B.\operatorname{Sen}C} + \frac{\operatorname{Cos}A.\operatorname{Cos}C + \operatorname{Sen}A.\operatorname{Sen}C}{\operatorname{Sen}A.\operatorname{Sen}C}$$

$$E = \frac{\operatorname{Cos}A.\operatorname{Cos}B}{\operatorname{Sen}A.\operatorname{Sen}B} + \cancel{\frac{\operatorname{Sen}A.\operatorname{Sen}B}{\operatorname{Sen}A.\operatorname{Sen}B}} + \frac{\operatorname{Cos}B.\operatorname{Cos}C}{\operatorname{Sen}B.\operatorname{Sen}C} + \cancel{\frac{\operatorname{Sen}B.\operatorname{Sen}C}{\operatorname{Sen}B.\operatorname{Sen}C}} + \frac{\operatorname{Cos}A.\operatorname{Cos}C}{\operatorname{Sen}A.\operatorname{Sen}C} + \cancel{\frac{\operatorname{Sen}A.\operatorname{Sen}C}{\operatorname{Sen}A.\operatorname{Sen}C}}$$

$$E = \operatorname{Cot}A.\operatorname{Cot}B + 1 + \operatorname{Cot}B.\operatorname{Cot}C + 1 + \operatorname{Cot}A.\operatorname{Cot}C + 1$$

$$E = 1 + 3$$

$$\therefore \mathbf{E = 4}$$

Clave: B

UNI 2009 – II:

Sean α, β, γ los ángulos de un triángulo, tal que $\text{Tan}\alpha + \text{Tan}\beta + \text{Tan}\gamma = 2007$. Entonces podemos afirmar que el valor de $1 + \text{Tan}\alpha.\text{Tan}\beta.\text{Tan}\gamma$ es

A) 2008 B) 2009 C) 2010 D) 2011 E) 2012

Resolución:

Entonces: $\alpha + \beta + \gamma = \pi$

Se cumple:

$$\underbrace{\text{Tan}\alpha + \text{Tan}\beta + \text{Tan}\gamma}_{2007} = \text{Tan}\alpha.\text{Tan}\beta.\text{Tan}\gamma$$

$$2007 + 1 = \text{Tan}\alpha.\text{Tan}\beta.\text{Tan}\gamma + 1$$

$$\therefore \mathbf{1 + Tan\alpha.Tan\beta.Tan\gamma = 2008}$$

Clave: A